StaTable Manual

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1 Introduction to StaTable

1.1 What It Is

StaTable (pronounced Stat-Table) is a compact program that runs on the family of IBM PC compatible computers under Microsoft Windows. It enables you to look up both tail areas and percentage points of 25 statistical distributions. You can read off a tail area or a percentage point of a random variate simply by typing a number into a pop-up window; no interpolation, no calculating errors, no hunting for tables. The distributions that you can look up are:

Continuous:

1. Normal
2. Uniform
3. Triangular
4. Beta
5. Gamma
6. Weibull
7. Chi-square
8. Non-central Chi-square
9. Student's T
10. Non-central Student's T
11. F-distribution
12. Non-central F
13. Gumbel
14. Logistic
15. Exponential
16. Lognormal
17. Pareto
18. Cauchy
19. Bivariate Normal

Discrete:

20. Binomial
21. Poisson
22. Geometric
23. Hypergeometric
24. Multinomial
25. Negative Binomial

In the Windows environment you can copy and paste numerical values between StaTable and your favorite spreadsheet, word processing, database, or statistical programs, through the Clipboard.

1.2 Algorithms and Testing

The algorithms used by StaTable are fully documented in Appendix D. StaTable has been extensively tested to ensure that the numbers it displays are accurate. Comparisons involving literally millions of values have been carried out against both standard books of tables and standard software. Some of the standards against which StaTable values have been tested are:

- *Biometrika Tables for Statisticians*, Vols. I & II by Pearson and Hartley (Charles Griffin, 1976).

- *Handbook of Mathematical Functions* by Abramowitz and Stegun (National Bureau of Standards, 1972).
- *Selected Tables in Mathematical Statistics* by Harter and Owen (American Mathematical Society, 1974).
- *Sample Size Choice* by Odeh and Fox (Marcel Dekker, 1975).
- *Tables of the Non-Central T-Distributions* by Resnikoff and Lieberman (Stanford University Press, 1957).
- *IMSL Sub-routines* by IMSL, Inc., 1989.
- *Applied Statistics Algorithms* by Griffiths and Hill, Royal Statistical Society (Ellis Horwood, 1985).
- Programs from *Transactions on Mathematical Software* and *Communications* of the Association for Computing Machinery.

For details concerning the testing procedures used to validate the StaTable output refer to Appendix D.

1.3 Installation for Windows Computers

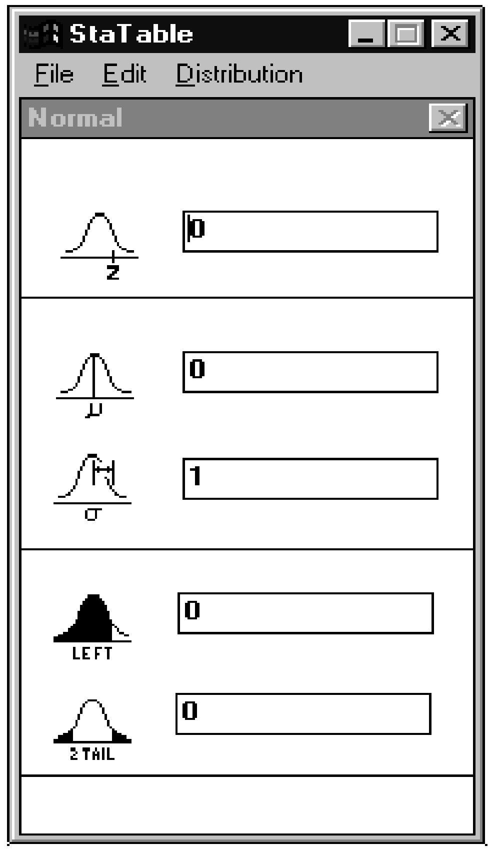
Installation of StatTable under Windows 3.1/95/NT consists of the following steps.

1. Run Windows as you normally would.
2. From the Windows Start menu or Program Manager's File menu, click on Run. You will see the Run dialog box.
3. Put the StaTable CD-ROM in the D: drive (or whichever drive you are installing from).
4. Type `D:\SETUP` and press *Enter*.
5. You will see a screen informing you that StaTable is loading the SetUp program. Follow the directions on your screen.
6. In the Target drive dialog box, type the drive and path where you wish to install StaTable. The default is `C:\STATABLE`. Click on OK.
7. When SETUP is complete, you will see the StaTable program group and icons.

2 StaTable for Windows

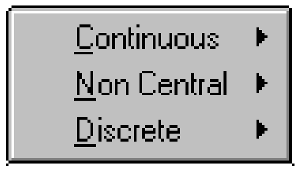
2.1 Starting StaTable

Open the StaTable program group from the Windows `Start -> Programs` menu (in Windows 95/NT) or the Program Manager (in Windows 3.1). Select the StaTable icon and double click on it. A distribution window labeled StaTable, similar to the one pictured below will open.

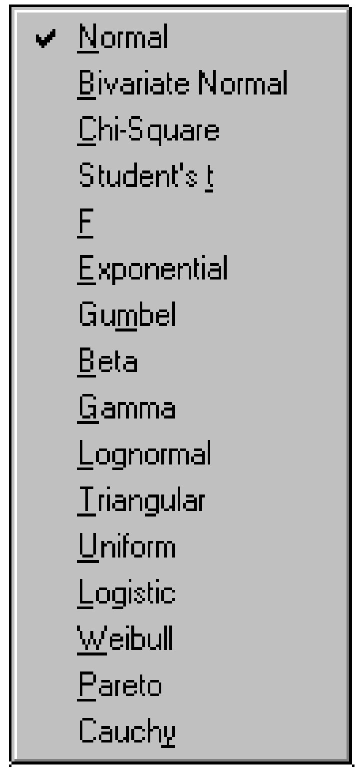


2.2 Selecting a Distribution

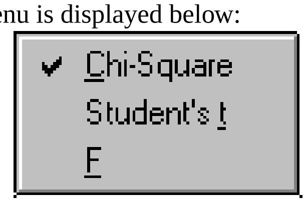
Notice that the window is set up to display the Normal distribution. If you want to look up any other distribution you must select it directly from one of the hierarchical menus under the Distribution menu. The distributions are organized into continuous, non-central and discrete categories, and the possible menu choices are pictured below:



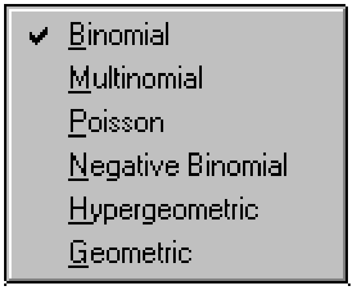
The `Continuous` sub-menu is displayed below:



The `Non-central` sub-menu is displayed below:



The `Discrete` sub-menu is displayed below:

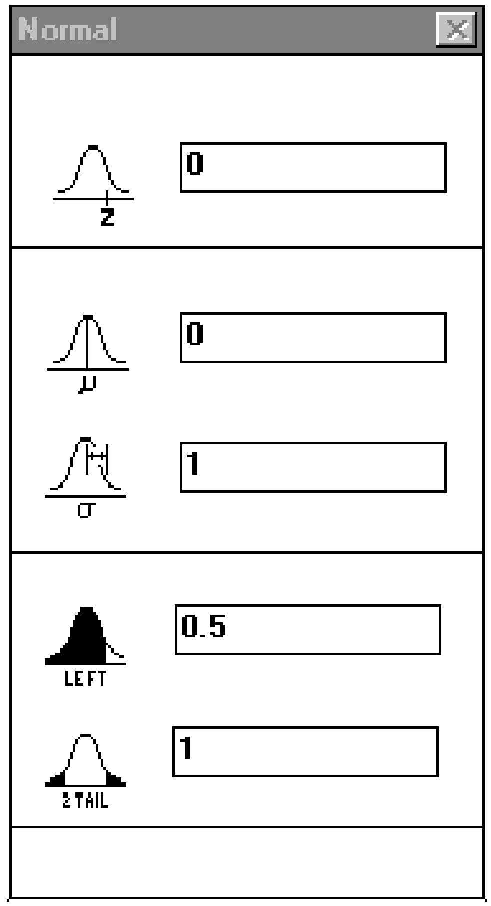


When you select the distribution you want, the contents of the window will change to reflect the parameters of that distribution. Appendix C gives the mathematical definition of each distribution in terms of its parameters.

2.3 Window Structure

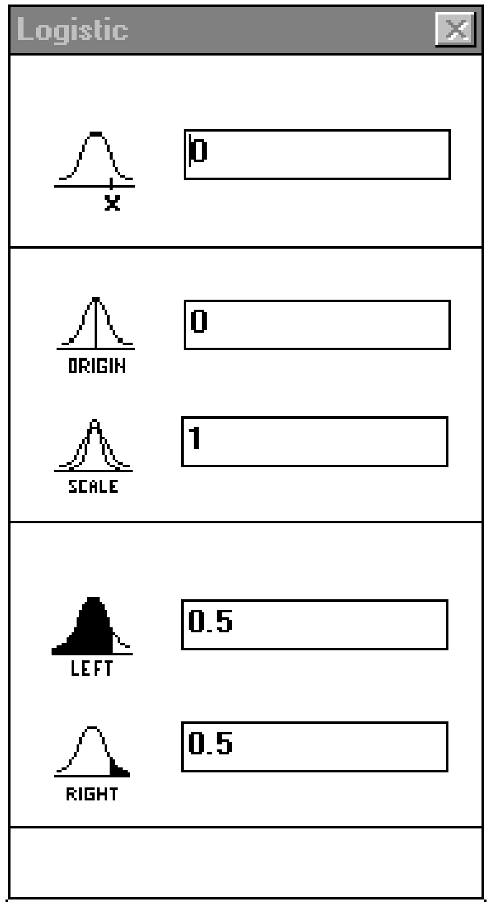
The structure of the window depends on whether the distribution is continuous or discrete.

The structure of the window for the continuous Normal distribution is shown below:

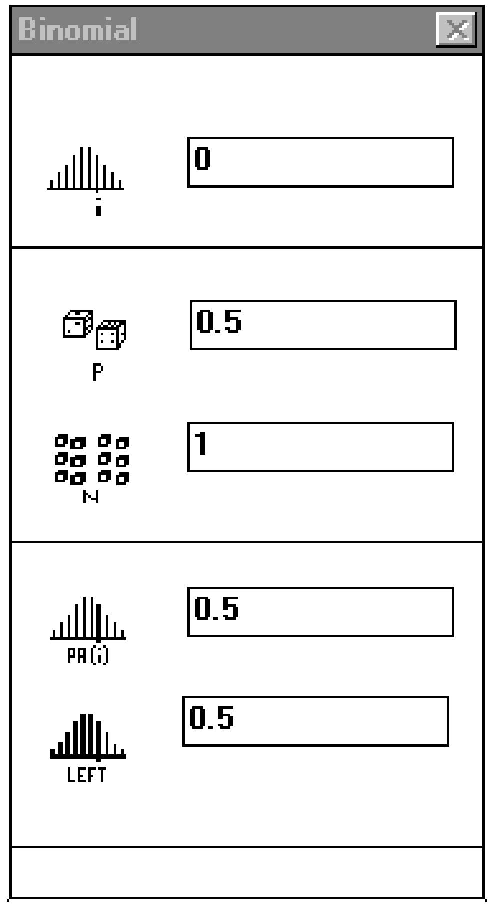


LEFT refers to the left tail (or cumulative) probability.

For the Normal and T distributions, **2 TAIL** gives the total area in both tails beyond the variate value. For all other continuous distributions, **RIGHT** ($= 1.0-$ **LEFT**) is displayed. The following picture of the Logistic distribution illustrates this.



For a discrete distribution the structure is only slightly different. In the bottom two rows, instead of the left and right tail probabilities, the point probability and the left tail probability (including the point) are displayed. The figure below illustrates the structure of the window for a discrete Binomial distribution.



Note the following:

The name of the distribution being currently displayed is shown at the top.

The window has a number of rows. (In the figure above there are five rows.) A window can have up to seven rows, depending on the distribution being displayed.

The rows are grouped into three main groups.

- The top group displays the random variate.
- The middle group consists of parameters of the distribution.
- The bottom group shows probabilities.

Each row has two parts. The right part consists of spreadsheet-like cells for entering and displaying numerical values. A vertical blinking cursor highlights one of these cells. This is the current input cell. Any number you type in from the keyboard will be entered into this cell. The cursor can be moved to any other row by using the `Tab` and `Shift_Tab` keys, or by clicking on the desired cell with the mouse.

The left part of each row displays a descriptive icon for the value displayed in the cell in the right part of that row.

EXAMPLES:

- The Normal distribution has five rows. The top row displays z, the Normal variate. The second and third rows show the mean and standard deviation. The last two rows give the left tail and two-tail values for the variate.
- The Chi-squared distribution has four rows. The top row displays the χ^2 variate. The second row shows the degrees of freedom and the last two rows display the left- and right-tail probabilities.
- The Poisson distribution has four rows. The top row displays i, the variate. The second row is the mean. The third row shows the point probability of i and the fourth row shows the left tail (cumulative probability) of i.
- The Binomial distribution has five rows. The top row displays i, the variate. The second row shows the probability of success for a single trial and the third row shows n, the number of trials. The fourth row displays the point probability of i successes and the fifth row shows the cumulative probability of i or fewer successes in n trials.
- The Bivariate Normal has four rows. The top two rows are for h and k, the variate values. The third row shows ρ, the correlation coefficient. The last row displays the joint probability of the x-variate exceeding h and the y-variate exceeding k.

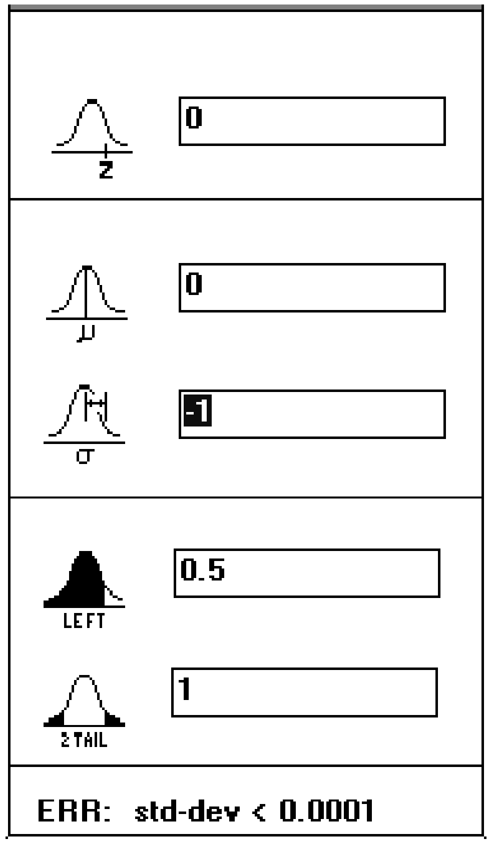
2.4 Looking Up Table Values

If you are looking up probabilities for a given variate value simply type the value(s) into the cell(s) in the variate row(s). Each cell entry is completed by pressing *Enter*, or by moving to another cell. Then enter the parameter values for the distribution if they are different from the currently displayed values. The desired probabilities will be computed and shown in the bottom group of probability cells.

If you are looking for the percentile point for a given probability, enter this value in the appropriate probability cell. The top-most cell will then display the required percentile point value.

EXAMPLES:

- Suppose that you want to find the probability that a chi-square random variable with 9 degrees of freedom has a value less than 18.5. Select *Chi-Square* from the continuous distribution sub-menu, to open a chi-square distribution window. Type in `18.5` in the top row, then move the cell cursor to the second row named **df** by pressing the `Tab` key once. Enter 9 in this cell. The probability you are seeking will be displayed in the row named **L tail** as .970204.
- Suppose that you want to look up the variate value for the .05 upper percentile of a Poisson distribution with mean = 10. Select *Poisson* from the Discrete Distribution sub-menu, to open the Poisson distribution window. Move the cursor to the second row named **mean** and enter 10. Now move the cursor down to the cell in the **L tail** row and enter .95. You will see the value 15 displayed as the value of i. This is the smallest value for which the probability of being less than or equal to i equals or exceeds .95. The window will also show you the point probability for $i = 15$ as .034718 in the row labeled **pr(i)** and a message will appear at the bottom of the window giving you the true probability of a variate value of 15 or less as .951260.
- Select *Normal* from the Continuous Distribution sub-menu to bring up the Normal Distribution window. Click the mouse near the left edge of the standard deviation cell in order to select this field and then prepend a minus sign to the number to make it a negative number. Since negative standard deviations are not allowed, when you hit *Enter*, StaTable will sound a beep, and put an error message at the bottom part of the distribution window, as shown below.



2.5 Commands

You can exit StaTable by double clicking the *System Button* at the upper left of the window or by selecting the `Exit` menu item under the File menu. This action will terminate the StaTable program and will release the utilized system resources.

At any time you can use the *Copy* and *Paste* editing commands from the Edit menu to transfer numbers to and from other Windows applications via the Clipboard.

A Ranges of Distribution Parameters

In the table below:

*	indicates the range [-99999 to 99999]
> 0	indicates the range [.0001 to 99999]
Parameter 1	is the parameter in the topmost row,
Parameter 2	is the parameter in the next row, and
Parameter 3	is the one in the last row of the parameter group of cells.

A Ranges of Distribution Parameters

Table of Parameter Ranges

Distribution	Parameter 1	Parameter 2	Parameter 3
Continuous			
Normal	*	> 0	
Bivariate Norm	[-1.0 to 1.0]		
Chi-square	[1 to 200]		
Non-central Chi-sq	[1 to 100]	[0.0 to 100.0]	
t	[1 to 500]		
Non-central t	[1 to 100]	[-150.0 to 150.0]	
F	[1 to 500]	[1 to 500]	
Non-central F	[1 to 500]	[1 to 500]	[0.0 to 500.0]
Exponential	[0.0001 to 1000.0]		
Extreme value	*	> 0	
Beta	[0.01 to 100.0]	[0.01 to 100.0]	
Gamma	> 0	[0.1 to 100.0]	
Lognormal	*	> 0	
Uniform	*	*	
Triangular	*	*	*
Logistic	*	> 0	
Weibull	*	> 0	[0.1 to 1000.0]
Pareto	> 0	[0.1 to 1000.0]	
Cauchy	*	> 0	
Discrete			
Binomial	[0.0001 to 0.9999]	[1 to 1000]	
Multinomial	[0.0001 to 0.9999]	[0.0001 to 0.9999]	[1 to 200]
Poisson	[0.0001 to 500]		
Neg Binomial	[1 to 100]	[0.01 to 0.99]	
Hypergeometric	[1 to 500]	[1 to 500]	[1 to 500]
Geometric	[0.01 to 0.99]		

Notes:

1. Variate values for all distributions range from -9999 to 9999.
2. You can enter values of Left-tail and Right-tail probabilities in the range [.001 to .999] for all distributions.

B Error Messages

ERR: **param** > **number** or ERR: **param** < **number**:

The number in the cell is outside the permissible range.

ERR: Incompatible Parameters:

The parameters of the distribution have incompatible values.

B *Error Messages*

C Definitions of Distributions

C.1 Continuous Distributions

C.1.1 Univariate

L Tail (the area in the left tail) $\equiv \int_{-\infty}^{x} f(u)du$ where $f(u)$ is the probability density function.

R Tail (the area in the right tail) $\equiv \int_{x}^{\infty} f(u)du = 1-$**L Tail**.

The definition of the left tail for each distribution in terms of the parameters displayed in the window is as follows:

DISTRIBUTION:	Normal	
VARIATE:	z	
PARAMETERS:	mean (m)	$-\infty < m < \infty$
	std dev(s)	$s > 0$
L TAIL:	$\int_{-\infty}^{z} \frac{e^{-(x-m)^2/2s^2}}{s\sqrt{2\pi}} dx$	

DISTRIBUTION:	chi^2	
VARIATE:	$chi^2(c)$	
PARAMETERS:	$df(d)$	$d = 1, 2, 3, \ldots$
L TAIL:	$\int_{0}^{c} \frac{(1/2)^{d/2} x^{d/2-1} e^{-x/2}}{\Gamma(d/2)} dx$ for $c \geq 0$	
	0 for $c < 0$	
COMMENTS:	Let $c \equiv z_1^2 + z_2^2 + \ldots + z_d^2$ where z_i are $i.i.d.$ normal random variables with mean = 0 and standard deviation = 1. Then c has a chi^2 distribution with d degrees of freedom.	

DISTRIBUTION:	t
VARIATE:	t

PARAMETERS: $df(d) \qquad d = 1, 2, 3, \ldots$

L TAIL: $\int_{-\infty}^{t} \frac{\Gamma[(d+1)/2]}{\sqrt{d\pi}\Gamma(d/2)} \cdot \frac{dx}{(1+x^2/d)^{(d+1)/2}}$

COMMENTS: Let $t \equiv z/\sqrt{c/d}$ where z, c are independent random variables such that z is normally distributed with mean = 0 and standard deviation = 1 and c has a chi^2 distribution with d degrees of freedom. Then t has a t- distribution with d degrees of freedom.

DISTRIBUTION: F

VARIATE: f

PARAMETERS: $dfnum(m) \qquad m = 1, 2, 3, \ldots$

$dfdenom(n) \qquad n = 1, 2, 3, \ldots$

L TAIL: $\int_{0}^{f} \frac{\Gamma[(m+n)/2]}{\Gamma(m/2)\Gamma(n/2)} (\frac{m}{n})^{m/2} \frac{x^{(m-2)/2}dx}{(1+mx/n)^{(m+n)/2}}$ for $f \geq 0$

0 for $f < 0$

COMMENTS: Let $f \equiv (c_1/m)/(c_2/n)$ where c_1, c_2 are independent chi^2 random variables with m and n degrees of freedom respectively. Then f has an F-distribution with parameters m, n.

DISTRIBUTION: Exponential

VARIATE: x

PARAMETERS: rate $(r) \qquad r > 0$

L TAIL: $\int_{0}^{x} re^{-ru}du$ for $x \geq 0$

0 for $x < 0$

DISTRIBUTION: Extreme-Value or Gumbel

VARIATE: x

PARAMETERS: location $(a) \qquad -\infty < a < \infty$

scale $(s) \qquad s > 0$

L TAIL: $\exp(-e^{-(x-a)/s})$

DISTRIBUTION: Beta

VARIATE: x

PARAMETERS: $a \qquad a > 0$

$b \qquad b > 0$

L TAIL:	$\int_0^x \frac{u^{a-1}(1-u)^{b-1}}{B(a,b)} du$ for $0 \leq x \leq 1$
	0 for $x < 0$
	1 for $x > 1$
DISTRIBUTION:	Gamma
VARIATE:	x
PARAMETERS:	scale (s) $s > 0$
	shape (h) $h > 0$
L TAIL:	$\int_0^x \frac{u^{h-1}}{s^h \Gamma(h)} e^{-u/s} du$ for $x \geq 0$
	0 for $x < 0$
DISTRIBUTION:	Lognormal
VARIATE:	x
PARAMETERS:	mu(m) $\infty < m < \infty$
	sigma (s) $s > 0$
L TAIL:	$\int_0^x \frac{1}{u\sigma\sqrt{2\pi}} \exp[-(\log u - m)^2/2s^2] du$ for $x > 0$
	0 for $x \leq 0$
COMMENTS:	Let $x \equiv e^z$ where z is normally distributed with mean = m and standard deviation = s. Then x has a lognormal distribution with parameters m, s.
DISTRIBUTION:	Uniform
VARIATE:	x
PARAMETERS:	low lim (a)
	up lim (b) $-\infty < a < b < \infty$
L TAIL:	$\int_a^x \frac{du}{(b-a)}$ for $a \leq x \leq b$
	0 for $x < a$
	1 for $x > b$
DISTRIBUTION:	Triangular
VARIATE:	t
PARAMETERS:	low lim (a)
	up lim (b)
	mode (m) $-\infty < a \leq m \leq b < \infty$
L TAIL:	$\int_a^t \frac{2}{(b-a)} \cdot \frac{(u-a)}{(m-a)} du$ for $a < t \leq m$

$1 - \int_t^b \frac{2}{(b-a)} \cdot \frac{(b-u)}{(b-m)} du$ for $m < t \leq b$
0 for $t \leq a$
1 for $t > b$

DISTRIBUTION:	Logistic
VARIATE:	x
PARAMETERS:	location (a) $\quad -\infty < a < \infty$
	scale (s) $\quad s > 0$
L TAIL:	$\frac{1}{[1+e^{-(x-a)/s}]}$

DISTRIBUTION:	Weibull
VARIATE:	x
PARAMETERS:	origin (a) $\quad -\infty < a < \infty$
	scale (s) $\quad s > 0$
	shape (h) $\quad h > 0$
L TAIL:	$1 - \exp\{-[(x-a)/s]^h\}$

DISTRIBUTION:	Pareto
VARIATE:	x
PARAMETERS:	origin (a) $\quad a > 0$
	shape (h) $\quad h > 0$
L TAIL:	$\int_a^x \frac{ha^h du}{u^{h+1}}$ for $x > a$
	0 for $x \leq a$

DISTRIBUTION:	Cauchy
VARIATE:	x
PARAMETERS:	location (a) $\quad -\infty < a < \infty$
	scale (s) $\quad s > 0$
L TAIL:	$\int_{-\infty}^x \frac{du}{s\pi\{1+[(u-a)/s]^2\}}$

DISTRIBUTION:	Non-Central chi^2
VARIATE:	$chi^2 \quad (c)$
PARAMETERS:	$df \quad (d) \quad d = 1, 2, \ldots$
	$n - ctrl(\ell) \quad \ell > 0$

L TAIL:	$e^{-\ell/2}\sum_{j=0}^{\infty}\frac{[\ell/2]^j}{j!}\cdot\int_0^c\frac{x^{\frac{d}{2}+j-1}e^{-x/2}dx}{2^{\frac{d}{2}+j}\Gamma(\frac{d}{2}+j)}$ for $c>0$ 0 for $c\le 0$
COMMENTS:	Let $c\equiv z_1^2+z_2^2+\ldots+z_d^2$ where z_i are $i.i.d.$ normal random variables with mean = m_i and standard deviation = 1. Then c has a non-central chi^2 distribution with parameters d and $\ell=m_1^2+m_2^2+\ldots+m_d^2$.

DISTRIBUTION:	Non-Central t
VARIATE:	t
PARAMETERS:	$df\ (d)\quad d=1,2,3\ldots$ $n-ctrl(\ell)\quad -\infty<\ell<\infty$
L TAIL:	$\frac{1}{2^{\frac{d}{2}-1}\Gamma(\frac{d}{2})}\int_0^{\infty}x^{d-1}e^{x^2/2}\int_0^{tx/\sqrt{d}}\frac{e^{-(u-\ell)^2/2}}{\sqrt{2\pi}}dudx$
COMMENTS:	Let $t\equiv z/\sqrt{c/d}$ where z,c are independent random variables such that z is normally distributed with mean = ℓ and standard deviation = 1 and c is chi^2 distributed with d degrees of freedom. Then t has a non-central $t-$distribution with parameters d and ℓ.

DISTRIBUTION:	Non-Central F
VARIATE:	f
PARAMETERS:	$df\ num(m)\quad m=1,2,\ldots$ $df\ denom(n)\quad n=1,2,\ldots$ $n-ctrl(\ell)\quad \ell>0$
L TAIL:	$e^{-\ell/2}\sum_{j=0}^{\infty}\frac{[\ell/2]^j}{j!}\cdot I_g(\frac{m}{2}+j,\frac{n}{2})$ for $f>0$ where $g\equiv mf/(n+mf)$ and $I_x(a,b)\equiv\int_0^x\frac{u^{a-1}(1-u)^{b-1}du}{B(a,b)}$ 0 for $f\le 0$
COMMENTS:	Let $f\equiv(c_1/m)/(c_2/n)$ where c_1,c_2 are independent random variables such that c_1 has a non-central chi^2 distribution with m degrees of freedom and non-centrality parameter = ℓ and c_2 has a (central) chi^2 distribution with n degrees of freedom. Then f has a non- central $F-$distribution with parameters m, n and ℓ.

C.1.2 Multivariate

DISTRIBUTION: Bivariate Normal
VARIATE: h, k are the values of the components of the bivariate normal vector.
PARAMETERS: rho (r) is the correlation coefficient, $-1 \leq r \leq 1$.
Pr$(> h, > k)$**:** This gives the probability that the first component of the random vector $> h$ and the second component of the random vector $> k$.

$$\Pr(> h, > k) \equiv \int_h^\infty \int_k^\infty \frac{1}{2\pi\sqrt{1-r^2}} e^{-\frac{(u^2-2ruv+v^2)}{2(1-r^2)}} dudv$$

C.2 Discrete Distributions

C.2.1 Univariate

i is the value of the random variate.
pr(i) is the probability of this value.
L tail is the cumulative probability that the random variable takes on a value less than or equal to **i** $\equiv \sum_{j=0}^{i}$ **pr(j)**

The definition of **pr(i)** for each distribution in terms of the parameters displayed in the window is given below.

DISTRIBUTION: Binomial
VARIATE: i
PARAMETERS: Pr suc (p) $0 < p < 1$
n $n = 1, 2, 3, \ldots$
pr(i): $\binom{n}{i} p^i (1-p)^{n-i}$ $i = 0, 1, \ldots, n$

DISTRIBUTION: Poisson
VARIATE: i
PARAMETERS: mean (m) $m > 0$
pr(i): $\frac{m^i e^{-m}}{i!}$ $i = 0, 1, 2 \ldots$

DISTRIBUTION:	Hypergeometric
VARIATE:	i_1
PARAMETERS:	$n_1 \quad n_1 = 1, 2, \ldots$
	$n_2 \quad n_2 = 1, 2, \ldots$
	$m \quad m = 1, 2, 3, \ldots$
	$m \leq n_1 + n_2$
pr(i):	$\dfrac{\binom{n_1}{i_1}\binom{n_2}{m - i_1}}{\binom{n_1 + n_2}{m}}$
	$\max(0, m - n_2) \leq i_1 \leq \min(m, n_1)$

DISTRIBUTION:	Geometric
VARIATE:	i
PARAMETERS:	Pr suc $(p) \quad 0 < p < 1$
pr(i):	$p(1 - p)^{i-1} \quad i = 1, 2, 3, \ldots$

DISTRIBUTION:	Negative Binomial
VARIATE:	i
PARAMETERS:	rqd suc (r)
	Pr suc $(p) \quad 0 < p < 1$
pr(i):	$\binom{i-1}{r-1} p^r (1 - p)^{i-r} \quad i = r, r + 1, r + 2, \ldots$

C.2.2 Multivariate

DISTRIBUTION:	Multinomial
VARIATE:	$i_1(i_2)$ is the number of type 1(2) items observed in the sample.
PARAMETERS:	p_1 probability of observing type 1 item
	p_2 probability of observing type 2 item
	n sample size
	$0 \leq p_1, p_2 < 1$ and $p_1 + p_2 \leq 1$
PROBABILITIES:	$\Pr(i_1, i_2) \equiv \frac{n!}{i_1!\, i_2!(n-i_1-i_2)!}(p_1)^{i_1}(p_2)^{i_2}(1 - p_1 - p_2)^{n-i_1-i_2}$

for $i_1 = 0, 1, 2, \ldots$
$i_2 = 0, 1, 2, \ldots$
$i_1 + i_2 \leq n$

$$\Pr(\leq i_1, i_2) \equiv \sum_{j_2=0}^{i_2} \sum_{j_1=0}^{i_1} \Pr(j_1, j_2)$$

D Algorithm Documentation and Output Validation

D.1 Algorithms used in StaTable

Below we document the algorithms used by StaTable for the continuous and discrete distributions. All floating point computations are in double precision (15 significant digits).

D.1.1 Continuous Distributions

Probability integrals for the following distributions are computed through C language implementations that closely parallel algorithms published in Applied Statistics and Communications of the Association for Computing Machinery (See Griffiths and Hill: Applied Statistics Algorithms, Royal Statistical Society, Ellis Horwood,1985):

1. Normal (Algorithm AS 66).
2. Bivariate Normal (Algorithm AS 76).
3. Chi-square (Algorithm AS 91).
4. Non-central Chi-square (Algorithm AS 170).
5. Student t (Algorithm AS 27; for degrees of freedom exceeding 500 the normal approximation is used).
6. Non-central t (Algorithms AS 5, ACM 291).
7. Beta (Algorithm AS 63).
8. Gamma (Algorithm AS 147).

Probability integrals for the following distributions are computed through C language implementations that evaluate simple closed form expressions (see, for example, Hastings and Peacock: Statistical Distributions, Wiley, 1974):

1. Exponential.
2. Extreme Value.
3. Uniform.
4. Triangular.
5. Logistic.
6. Weibull.
7. Pareto.
8. Cauchy.

Probability integrals for the following distributions are computed through C language implementations which make a single call to one of the above routines after an appropriate simple transformation:

1. Lognormal (calls Normal).
2. F (calls Beta).

Probability integrals for the following distribution are computed through a series expansion using incomplete beta function ratios (See Johnson and Kotz: Distributions in Statistics-Continuous Univariate Distributions 2, Wiley, 1970 p.192):

1. Non-central F.

For all continuous distributions the percentile points are computed using a bisection method coupled with the above probability integral routines. The only exception is the bivariate normal for which the percentile point is not calculated since a unique percentile point does not exist.

D.1.2 Discrete Distributions

Cumulative probabilities for all the discrete distributions are computed as sums of the individual point probabilities. The point probabilities are themselves computed using simple closed form expressions (see, for example, Hastings and Peacock: Statistical Distributions, Wiley, 1974).

For all discrete distributions the percentile points are computed by calculating the cumulative probability by summation as indicated above and stopping when the sum exceeds the required probability.

The only exception to the above algorithms is the multinomial for which the cumulative distribution is computed by making multiple calls to the binomial routine with appropriate parameters. For the multinomial the percentile point is not calculated since a unique percentile point does not exist.

D.2 Validating the StaTable Output

The following distributions and their percentile points were tested by generating between 100,000 and 300,000 random sets of parameter values drawn uniformly from their valid parameter ranges as defined in Appendix **??**. The number of sets generated depended on the number of parameters and the time and complexity of the computation. These random sets of values were fed to IMSL or SAS routines to create files of input and output values. The routines in StaTable computed for each input value set an output value (either a probability integral or percentile point, depending on the routine being tested). This output value was compared for an exact match upto the digits displayed by StaTable with the corresponding values as computed by IMSL or SAS. All the millions of tested values matched exactly.

1. Normal and its inverse.
2. Bivariate Normal.
3. Chi-square and its inverse.
4. Non-central Chi-square and its inverse.
5. Student's t and its inverse.
6. Non-central t and its inverse.
7. F and its inverse.
8. Non-central F and its inverse.
9. Beta and its inverse.
10. Gamma and its inverse.
11. Lognormal.
12. Binomial.
13. Poisson.
14. Negative Binomial.
15. Hypergeometric.

The following distributions were tested by generating 20 files each with 5,000 random sets of parameter values drawn uniformly from their valid parameter ranges as defined in Appendix A. Each file was used as input to StaTable routines which wrote out ASCII files of input and output sets. These ASCII files were imported into a Lotus 1-2-3 spreadsheet with the corresponding formulas for probability integrals and percentile points. The values were tested for equality upto the digits displayed by StaTable using spreadsheet formulas. All the millions of tested values matched exactly.

1. Exponential.
2. Extreme Value.
3. Uniform.
4. Triangular.
5. Logistic.
6. Weibull.
7. Pareto.
8. Cauchy.
9. Geometric.

10. Multinomial.

In addition values were checked for several thousand cases by selecting values near the extremes of the tabulations for both percentile points and cumulative probabilities for distributions from:

1 Biometrika Tables for Statisticians, volumes I & II by Pearson and Hartley, Charles Griffin, 1976 (Normal, Chi-square, Student's t, Beta, F, Binomial, Poisson, Non-central Chi-square, Non-central t, and Non-central F).
2 Handbook of Mathematical Functions by Abramowitz and Stegun, National Bureau of Standards, 1972 (Normal, Bivariate Normal, Beta, F and Student's t).
3 Tables of the Non-central t-Distributions by Resnikoff and Lieberman, Stanford University Press, 1957.
4 Selected Tables in Mathematical Statistics by Harter and Owen, Vol .II American Mathematical Society, 1974 (Non-central t and Doubly Non-central F).

These values were entered into StaTable and the results were compared. All cases matched upto the digits displayed by StaTable.

Finally a number of cross-checking runs each involving 100,000 to 300,000 cases were made comparing two different distributions within StaTable exploiting known and provable relationships between the distributions. The relationships exploited were (see reference [**2**] above):

1. Between Non-central F with 1 degree of freedom and non-central t. (Note: there is an error in the formula given in Abramowitz and Stegun in formula 26.6.19).
2. Between Negative Binomial and Binomial.
3. Between Gamma and Chi-square.
4. Between Exponential and Gamma.
5. Between Student t and Beta.
6. Between Poisson and Chi-square.
7. Between Binomial and Beta.

All the millions of tested values matched exactly upto the digits displayed by StaTable.